AP* Calculus Review

Position, Velocity, and Acceleration

Teacher Packet

Session Notes

Suppose an object is moving along a straight line, such as the *x*-axis, so that its position *x*, as a function of time *t*, on that line is given by $y = x(t)$.

Average velocity of the object over the time interval t to $t + \Delta t$ is given by $\frac{x(t+\Delta t) - x(t)}{\Delta t}$, or $\frac{\text{change in position}}{\text{change in time}}$.

Instantaneous velocity of the object is the derivative of the position function $x(t)$ with respect to time. $v(t) = x'(t)$

Speed is the absolute value of the velocity. $Speed = |v(t)| = \left|\frac{dx}{dt}\right|$.

Acceleration is the derivative of velocity with respect to time. $a(t) = v'(t) = x''(t)$

$\int v(t)\,dt = x(t) + c$,

$\int a(t)\,dt = v(t) + c$

Total distance traveled from time $t = t_1$ *to* $t = t_2$ is given by

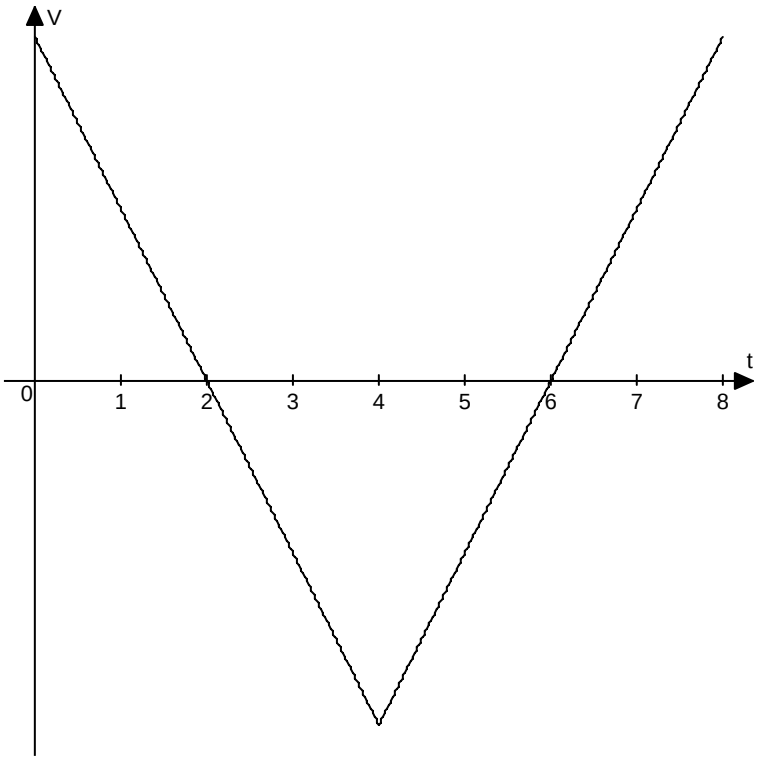
$TDT = \int_{t_1}^{t_2} |v(t)|\,dt$.

Speeding Up or Slowing Down

If the velocity and acceleration have the same sign (both positive or both negative), then speed is increasing. If an object's velocity is -40 miles per hour and the object accelerates -10 miles per hour per hour, the object is speeding up.

If the velocity and acceleration are opposite in sign (one is positive and the other is negative), then speed is decreasing. If an object's velocity is -40 miles per hour and the object accelerates 10 miles per hour per hour, the object is slowing down.

Sign Convention: When the object is moving in the right direction or moving upward then the velocity is positive (Graph of velocity vs. time is above the t axis). When the object is moving in the left direction or moving downward then the velocity is negative. A graph of velocity vs. time is shown below.



$0 < t < 2$	$v(+), a(-)$	Object is slowing down
$2 < t < 4$	$v(-), a(-)$	Object is speeding up
$4 < t < 6$	$v(-), a(+)$	Object is slowing down
$6 < t < 8$	$v(+), a(+)$	Object is speeding up

What you need to know about motion along the *x*-axis:

When you see…	**Think…**
Initially	$t = 0$
At rest	$v(t) = 0$
At the origin	$x(t) = 0$
Velocity is positive	Particle is moving right (or up)
Velocity is negative	Particle is moving left (or down)
Average velocity (Given $x(t)$)	Change in position divided by change in time
Average velocity (Given $v(t)$)	$\frac{1}{b-a}\int_a^b v(t)\,dt$ (The average value of the velocity function.)
Instantaneous velocity	Velocity at an exact moment
Positive acceleration	Velocity is increasing
Negative acceleration	Velocity is decreasing
Speed	$\lvert v(t)\rvert$
Total Distance Traveled	$\int_a^b \lvert v(t)\rvert\,dt$

Multiple Choice

1. The velocity of a particle moving on a line at time t is $v = 5t^{\frac{2}{3}} + 6t$. How many meters did the particle travel from $t = 1$ to $t = 8$?

(A) $-\frac{10}{3}$ (B) 224 (C) 279

(D) 282 (E) 533

2. The position of the particle is given as $x(t) = \cos(3t) - \sin(4t)$. Find the acceleration at $t = 0$.

(A) -9 (B) 0 (C) 1

(D) 2 (E) 16

3. A particle moves along the *x*-axis with acceleration at any time *t* given as $a(t) = 3t^2 + 4t + 6$. If the particle's initial velocity is 10 and its initial position is 2, what is the position function?

(A) $x(t) = \frac{1}{4}t^4 + \frac{2}{3}t^3 + 3t^2 + 12$ (B) $x(t) = \frac{1}{4}t^4 + \frac{2}{3}t^3 + 3t^2 + 10t + 2$

(C) $x(t) = 3t^4 + t^3 + t^2 + 10t + 2$ (D) $x(t) = 3t^4 + t^3 + t^2 + 2$

(E) $x(t) = \frac{1}{4}t^4 + \frac{2}{3}t^3 + 3t^2 + 2$

4. If the position of an ant traveling along a horizontal path at time *t* is $3t^2 + 1$, what is the ant's average velocity from $t = 1$ to $t = 6$?

(A) $\frac{1}{21}$ (B) 6 (C) $\frac{109}{6}$

(D) 21 (E) 220

The following information applies to problems 5, 6 and 7.

A bottle rocket is shot upward from a 10 foot stand with velocity $v(t) = 50 - 1.6t$.

5.GC What is the position of the bottle rocket after 2 seconds?

(A) 46.8 ft (B) 56.8 ft (C) 96.8 ft

(D) 103.6 ft (E) 106.8 ft

6.GC When will the bottle rocket hit the ground?

(A) $t = 0$ (B) $t = 8.66$ (C) $t = 31.448$

(D) $t = 62.5$ (E) $t = 62.699$

7.GC After 3 seconds the rocket is

(A) falling at an increasing rate
(B) rising at an increasing rate
(C) rising at a decreasing rate
(D) falling at a decreasing rate
(E) rising a constant rate

8. The position of the particle traveling along a straight line is $x(t) = t^3 - 9t^2 + 15t + 3$. On the interval $t = 0$ to $t = 10$, when is the particle farthest to the left?

(A) $t = 0$ (B) $t = 1$ (C) $t = 3$

(D) $t = 5$ (E) $t = 10$

9.GC Choose the integral expression that would result in the total distance traveled on the interval [0, 3] if the velocity is given by $v(t) = e^t - 6$.

(A) $\int_0^{\ln 6} (e^t - 6)\,dt + \int_{\ln 6}^{3} (e^t - 6)\,dt$

(B) $\int_3^{\ln 6} (e^t - 6)\,dt - \int_{\ln 6}^{0} (e^t - 6)\,dt$

(C) $\int_{\ln 6}^{3} (e^t - 6)\,dt - \int_0^{\ln 6} (e^t - 6)\,dt$

(D) $\int_0^{\ln 6} (e^t - 6)\,dt - \int_{\ln 6}^{3} (e^t - 6)\,dt$

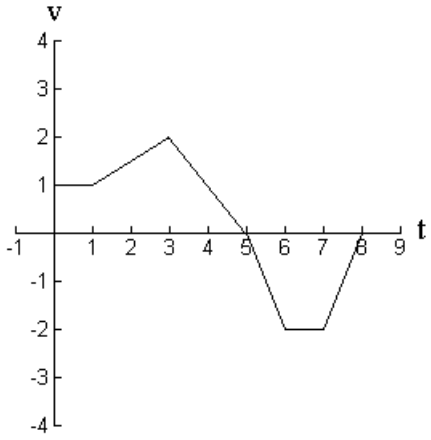
(E) $\int_0^{3} (e^t - 6)\,dt$

10.GC When two particles start at the origin with velocities $v(t) = 4\cos t$ and $v(t) = 4\sin t$, how many times in the interval $[0, 2\pi]$ will their speeds be equal?

(A) 2 (B) 3 (C) 4

(D) 5 (E) 6

Free Response 1 – No Calculator



The graph given above is $y = v(t)$, the velocity of an object moving on a line over the time interval [0, 8]. At $t = 0$ the position of the object is 5.

(a) When is the object at rest?

(b) Evaluate $\int_1^6 v(t)\,dt$. Explain the meaning of the result.

(c) What is the position of the object at $t = 5$?

(d) Find the total distance traveled over [0, 8].

(e) At $t = 2$, is the object speeding up or slowing down? Explain your answer.

Free Response 2 – Calculator allowed

t (hours)	v (miles per hour)
0	0
0.25	10.3
0.5	13.1
0.75	12.8
1	16.2
1.25	20.1
1.5	20.2
1.75	14.3
2	9.6

The table represents data collected in an experiment on a new type of electric engine for a small neighborhood vehicle (*i.e.*, one that is licensed for travel on roads with speed limits of 35 mph or less).

The readings represent velocity, in miles per hour, taken in 15-minute intervals on a 2 hour trip.

(a) What is the average acceleration over [0.25, 0.75]? Include units.

(b) What is the meaning of $\frac{1}{b-a}\int_a^b v(t)\,dt$ if $a = 0.75$ and $b = 2$? Include units in your answer.

(c) Use a midpoint Riemann sum with four subintervals to approximate $\int_0^2 v(t)\,dt$.

(d) At the end of the two hours the vehicle is 35 miles from a source for recharging the battery. Assuming that the vehicle can travel 75 miles on a single charge, can the vehicle get back to the source (without being towed or pushed)? Explain your answer.

Free Response 3 – No Calculator

Let $v(t) = \frac{1}{\pi} + \sin 3t$ represent the velocity of an object moving on a line. At $t = \frac{\pi}{3}$, the position is 4.

(a) Write the acceleration function.

(b) Write the position function.

(c) At $t = \frac{\pi}{4}$ is the object speeding up or slowing down? Explain your answer.

(d) On the interval $\left[\frac{\pi}{2}, \pi\right]$, what is the velocity when the acceleration is 3?